Mathematical Formulae for Electrical and Computer Engineers

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Chapter 1

Algebra

1.1 Factors

$$
\begin{aligned}
(a+b)^2 &= a^2+2ab+b^2 \\
(a-b)^2 &= a^2-2ab+b^2 \\
(a+b+c)^2 &= a^2+b^2+c^2+2bc+2ca+2ab \\
a^2-b^2 &= (a+b)(a-b) \\
a^3+b^3 &= (a+b)(a^2-ab+b^2) \\
a^3-b^3 &= (a-b)(a^2+ab+b^2) \\
(a+b)^3 &= a^3+3a^2b+3ab^2+b^3 \\
(a-b)^3 &= a^3-3a^2b+3ab^2-b^3 \\
a^3+b^3+c^3-3abc &= (a+b+c)(a^2+b^2+c^2-bc-ca-ab)
\end{aligned}
$$

1.2 Partial fractions

Provided that the numerator $f(x)$ is of less degree than the relevant denominator, the following identities are typical examples of partial fraction expansion :

- only distinct linear factors :

$$\frac{f(x)}{(x+\alpha_1)\dots(x+\alpha_n)} = \frac{A_1}{x+\alpha_1} + \dots + \frac{A_n}{x+\alpha_n}$$

- repeated linear factors :

$$\frac{f(x)}{(x+\alpha)^n} = \frac{A_1}{x+\alpha} + \ldots + \frac{A_n}{(x+\alpha)^n}$$

- a quadratic factor :

$$\frac{f(x)}{(\alpha_1 x^2 + \beta_1 x + \gamma_1)(\alpha_2 x^2 + \beta_2 x + \gamma_2)} = \frac{Ax+B}{(\alpha_1 x^2 + \beta_1 x + \gamma_1)} + \frac{Cx+D}{(\alpha_2 x^2 + \beta_2 x + \gamma_2)}$$

1.3 Law of indices

$$\begin{aligned}
a^m \times a^n &= a^{m+n} \\
\frac{a^m}{a^n} &= a^{m-n} \\
(a^m)^n &= a^{mn} \\
a^{-n} &= \frac{1}{a^n} \\
a^0 &= 1 \\
\sqrt[n]{a^m} &= a^{m/n} \\
\sqrt[n]{ab} &= \sqrt[n]{a} \times \sqrt[n]{b} \\
\sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}}
\end{aligned}$$

1.4 Logarithm

Definition : If $y = a^x$, then

$$\begin{aligned}
x &= \log_a y \qquad \text{(“log y to base a”)} \\
y &= a^{\log_a x}
\end{aligned}$$

Logarithmic rules

$$\begin{aligned}
\log(a \times b) &= \log a + \log b \\
\log\left(\frac{a}{b}\right) &= \log a - \log b \\
\log(x^a) &= a \log x \\
\log{(y)}^{1/n} &= \frac{\log y}{n} \\
\log_b a &= \frac{\log_c a}{\log_c b} \qquad \text{(Change of base)} \\
\log(a + jb) &= \log \sqrt{a^2 + b^2} + j \tan^{-1} \frac{b}{a}
\end{aligned}$$

1.5 Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac > 0$, then $ax^2 + bx + c = 0$ yields two real and distinct roots.
- If $b^2 - 4ac = 0$, then $ax^2 + bx + c = 0$ yields two real and equal roots.
- If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ yields a pair of complex conjugate roots.

1.6 Arithmetic Progression

If a = first term, d = common difference, n = number of terms and l = last term, then the arithmetic progression is

$$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)d, \ldots, l$$

$$\begin{aligned}
\text{Sum of } n \text{ terms} &= \frac{n}{2}\left[2a + (n - 1)d\right] \\
&= \frac{n}{2}(a + l)
\end{aligned}$$

1.7 Geometric Progression

If a = first term, r = common ratio and n = number of terms, then the geometric progression is :

$$a, ar, ar^2, ar^3, \ldots, ar^{n-1}, \ldots$$

$$\text{Sum of } n \text{ terms } = \begin{cases} \frac{a(r^n-1)}{r-1}, & r > 1 \\ \frac{a(1-r^n)}{1-r}, & r < 1 \end{cases}$$

Sum to infinity when $r < 1 = \dfrac{a}{1-r}$

1.8 Series

Binomial series:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \qquad (\text{valid for } -1 < x < 1)$$

Exponential series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$$

Taylor's expansion:

$$f(x+a) = f(x) + af'(x) + \frac{a^2}{2!}f''(x) + \frac{a^3}{3!}f'''(x) + \ldots$$

Maclaurin's form:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$

Chapter 2

Geometry

Equation of a straight line joining (x_1, y_1) **and** (x_2, y_2) :

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\text{or} \quad y - y_1 = m(x - x_1)$$

$$y = mx + c \qquad \text{where } m = \text{ gradient and } c = y\text{-intercept}$$

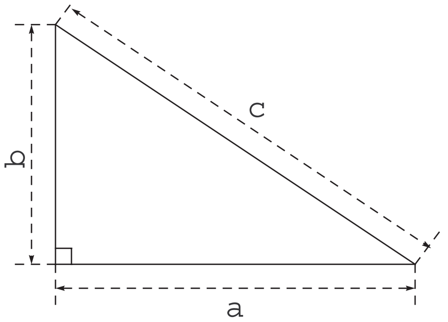
Equation of a circle, center at (x_0, y_0)**, radius r** :

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Equation of an ellipse, center at (x_0, y_0)**, semi-axes a and b** :

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

Pythagoras Theorem :

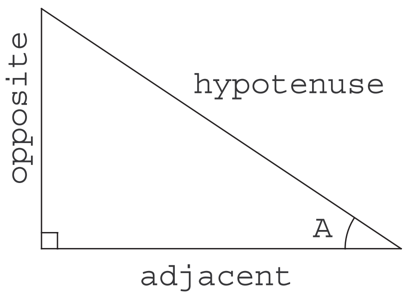


$$c^2 = a^2 + b^2$$

Chapter 3

Trigonometry

3.1 Definitions



$$\begin{aligned}
\sin A &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\cos A &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\tan A &= \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin A}{\cos A} \\
\text{cosec } A &= \frac{1}{\sin A} = \frac{\text{hypotenuse}}{\text{opposite}} \\
\sec A &= \frac{1}{\cos A} = \frac{\text{hypotenuse}}{\text{adjacent}} \\
\cot A &= \frac{1}{\tan A} = \frac{\text{adjacent}}{\text{opposite}}
\end{aligned}$$

3.2 Signs and variations of trigonometric functions

Quadrant	$\sin A$	$\cos A$	$\tan A$	$\cot A$	$\sec A$	cosec A
I	$+$ 0 to 1	$+$ 1 to 0	$+$ 0 to ∞	$+$ ∞ to 0	$+$ 1 to ∞	$+$ ∞ to 1
II	$+$ 1 to 0	$-$ 0 to -1	$-$ $-\infty$ to 0	$-$ 0 to $-\infty$	$-$ $-\infty$ to -1	$+$ 1 to ∞
III	$-$ 0 to -1	$-$ -1 to 0	$+$ 0 to ∞	$+$ ∞ to 0	$-$ -1 to $-\infty$	$-$ $-\infty$ to -1
IV	$-$ -1 to 0	$+$ 0 to 1	$-$ $-\infty$ to 0	$-$ 0 to $-\infty$	$+$ ∞ to 1	$-$ -1 to $-\infty$

3.3 Identities

$$\begin{aligned}
\sin^2 A + \cos^2 A &= 1 \\
1 + \tan^2 A &= \sec^2 A \\
\cot^2 A + 1 &= \text{cosec}^2 A \\
\text{cosec}^2 A &= 1 + \cot^2 A \\
\sin(-A) &= -\sin A \\
\cos(-A) &= +\cos A \\
\tan(-A) &= -\tan A
\end{aligned}$$

3.4 Compound angle addition and subtraction formula

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\end{aligned}$$

3.5 Double angles

$$\begin{aligned}
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} \\
\sin 3A &= 3 \sin A - 4 \sin^3 A \\
\cos 3A &= 4 \cos^3 A - 3 \cos A \\
\tan 3A &= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}
\end{aligned}$$

3.6 Products of sines or cosines into sums or differences

$$\begin{aligned} 2\sin A\cos B &= \sin(A+B)+\sin(A-B) \\ 2\cos A\sin B &= \sin(A+B)-\sin(A-B) \\ 2\cos A\cos B &= \cos(A+B)+\cos(A-B) \\ 2\sin A\sin B &= \cos(A-B)-\cos(A+B) \end{aligned}$$

3.7 Sums or differences of sines or cosines into products

$$\begin{aligned} \sin A+\sin B &= 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \\ \sin A-\sin B &= 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \\ \cos A+\cos B &= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \\ \cos A-\cos B &= 2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \end{aligned}$$

3.8 Polar Form

$$a\sin x+b\cos x=R\sin(x+\phi)$$

where $R=\sqrt{a^2+b^2}$

$\theta=\tan^{-1}\frac{b}{a}$

$\sin\phi=\frac{b}{R}$

$\cos\phi=\frac{a}{R}$

3.9 Complex exponent forms : imaginary form

$\cos x = \frac{e^{jx}+e^{-jx}}{2}$ $\qquad e^{jx} = \cos x + j \sin x$

$\sin x = \frac{e^{jx}-e^{-jx}}{2j}$ $\qquad e^{-jx} = \cos x - j \sin x \; j = \sqrt{-1}$

Chapter 4

Hyperbolic Function

4.1 Definitions

$$\sinh A = \frac{e^A - e^{-A}}{2} \qquad \text{cosech } A = \frac{1}{\sinh A} = \frac{2}{e^A - e^{-A}}$$

$$\cosh A = \frac{e^A + e^{-A}}{2} \qquad \text{sech } A = \frac{1}{\cosh A} = \frac{2}{e^A + e^{-A}}$$

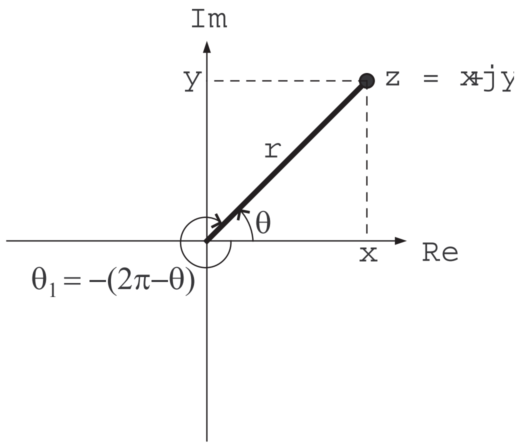
$$\tanh A = \frac{e^A - e^{-A}}{e^A + e^{-A}} \qquad \coth A = \frac{1}{\tanh A} = \frac{e^A + e^{-A}}{e^A - e^{-A}}$$

4.2 Identities

$$\begin{aligned} \cosh^2 A - \sinh^2 A &= 1 \\ 1 - \tanh^2 A &= \text{sech}^2 A \\ \coth^2 A - 1 &= \text{cosech}^2 A \end{aligned}$$

Chapter 5

Complex Numbers



$$\begin{aligned} z &= x + jy \quad \text{where } j = \sqrt{-1} \\ &= r(\cos\theta + j\sin\theta) \\ &= r\angle\theta \\ &= re^{j\theta} \\ r &= |z| \;=\; \sqrt{x^2+y^2} \\ \theta &= arg\ z \;=\; \theta_1 \\ &= \tan^{-1}\frac{b}{a} = \sin^{-1}\frac{y}{|z|} = \cos^{-1}\frac{x}{|z|} \end{aligned}$$

Addition : $(a+bj)+(c+dj)=(a+c)+(b+d)j$

Subtraction : $(a+bj)-(c+dj)=(a-c)+(b-d)j$

Multiplication : $z_1z_2 = r_1r_2\angle(\theta_1+\theta_2)$

Division : $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\angle(\theta_1-\theta_2)$

De Moivre's theorem : $[r\angle]^n = r^n\angle n\theta = r^n(\cos n\theta + j\sin n\theta)$

Chapter 6

Differentiation

Function	Derivative	Function	Derivative
x^n	nx^{n-1}	$\frac{1}{x}$	$-\frac{1}{x^2}$
$\log_e n$	$\frac{1}{x}$	$\log_a x$	$\frac{1}{x \log_e a}$
e^{cx}	ce^{cx}	$a^x\ (a > 0)$	$a^x \log_e a$
$\sin x$	$\cos x$	$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$	$\text{cosec } x$	$-\text{cosec } x \cot x$
$\sec x$	$\sec x \tan x$	$\cot x$	$-\text{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$		

Product rule :

When $y = uv$ and u and v are functions of x, then

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Quotient rule :

When $y = \frac{u}{v}$ and u and v are functions of x, then

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain rule :

If u is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Implicit differentiation :

$$\frac{d}{dx}[f(y)] = \frac{d}{dy}[f(y)] \times \frac{dy}{dx}$$

Maximum and minimum values :

If $y = f(x)$, then the stationary points are found by solving $\frac{dy}{dx} = 0$.

Let a solution of $\frac{dy}{dx} = 0$ be $x = a$. If the value of $\frac{d^2y}{dx^2}$ when $x = a$ is :

- positive, the point is a minimum point.
- negative, the point is a maximum point.
- zero, the point is a point of inflexion (saddle point).

Chapter 7

Integration

Function	Integral	Function	Integral
$x^n \ (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\frac{1}{x}$	$\log_e \lvert x \rvert$
e^x	e^x	$\frac{f'(x)}{f(x}$	$\log_e \lvert f(x) \rvert$
$\sin x$	$-\cos x$	$\cos x$	$\sin x$
$\tan x$	$\log_e \lvert \sec x \rvert$	$\text{cosec } x$	$\log_e \lvert \tan \frac{x}{2} \rvert$
$\sec x$	$\log_e \lvert \tan(\frac{\pi}{4} + \frac{x}{2}) \rvert$	$\cot x$	$\log_e \lvert \sin x \rvert$
$\frac{1}{a^2-x^2} \ (\lvert x \rvert < a)$	$\frac{1}{2a} \log_e \frac{a+x}{a-x}$	$\frac{1}{a^2+x^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a}$

Integration by parts : If u and v are both functions of x, then

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Chapter 8

Differential Equations

8.1 First order differential equations

1. $\frac{dy}{dx} = f(x)$ or $P\frac{dy}{dx} + Q = 0$, P and Q being functions of x only.

   $P\frac{dy}{dx} + Q = 0$ can be written as $\frac{dy}{dx} = -\frac{Q}{P} = f(x)$

   Solution : $y = \int f(x)\, dx$

2. If $\frac{dy}{dx} = f(y)$ or $P\frac{dy}{dx} + Q = 0$, P and Q being functions of y only.

   $P\frac{dy}{dx} + Q = 0$ can be written as $\frac{dy}{dx} = -\frac{Q}{P} = f(y)$

   Solution : $\int dx = \int \frac{dy}{f(y)}$

3. Separation of variables : $\frac{dy}{dx} = f(x) \cdot f(y)$

   Solution : $\int \frac{dy}{f(y)} = \int f(x)\, dx$

4. Homogeneous first order differential equation : $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

   Solution :

   - Introduce the new independent variable $v = \frac{y}{x}$.
   - Then, $y = vx$ and $\frac{dy}{dx} = v(1) + x\frac{dv}{dx}$ by the product rule.
   - Substitute $y = vx$ and $dydx = v(1) + x\frac{dv}{dx}$ in $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ to obtain a separable differential equation. Solve the separable differential equation.
   - Solution is obtained by replacing v by $\frac{y}{x}$

5. Linear first order differential equation : $\dfrac{dy}{dx} + P(x)y = Q(x)$

   Solution :

   - Determine the integrating factor (I.F.) : $e^{\ P(x)dx}$
   - Substitute the I.F. into the equation : $ye^{\ P(x)dx} = \int Q(x)e^{\ P(x)dx}\,dx$,
   - $ye^{\ P(x)dx}$ will be an exact differential and $\int Q(x)e^{\ P(x)dx}\,dx$ can be integrated

8.2 Second order differential equations

Linear homogeneous second order differential equations

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

- Form the characteristic equation : $am^2 + bm + c = 0$
- If the roots of the complementary equation are
  - *real and distinct* i.e. $m = \alpha$ and $m = \beta$, then the general solution is

    $$y = Ae^{\alpha x} + Be^{\beta x}$$

  - *real and equal* i.e. $m = \alpha$ twice, then the general solution is

    $$y = e^{\alpha x}(A + Bx)$$

  - *complex* i.e. $m = \alpha \pm j\beta$, then the general solution is

    $$y = e^{\alpha x}(A\sin\beta x + B\cos\beta x)$$

- Constants A and B are determined from initial conditions.

Linear non-homogeneous second order differential equations

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

The total solution of this type of differential equation is made up of :

- The general solution of the homogeneous second order differential equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

- The particular integral, which depends on $f(x)$. The following table lists the trial particular integral to use for various $f(x)$.

$f(x)$	Trial particular integral
$f(x) = \text{constant}$	• $y = k$ • $y = kx$ when general solution contains a constant
$f(x) = L + Mx + Nx^2 + \ldots$	• $y = a + bx + cx^2 + \ldots$
$f(x) = Ae^{\alpha x}$	• $y = ke^{\alpha}x$ • $y = kxe^{\alpha x}$ when general solution contains $e^{\alpha x}$ • $y = kx^2e^{\alpha x}$ when general solution contains $xe^{\alpha x}$, and so on
$f(x) = \alpha \sin px + \beta \cos x$ α or β may be zero	• $y = a \sin px + b \cos px$ • $y = x(a \sin px + b \cos px)$ when general solution contains $\sin px$ and/or $\cos px$

Chapter 9

Laplace Transform

9.1 Definition

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,\mathrm{d}t$$

9.2 Laplace Transform Pairs

$f(t)$	$\Leftrightarrow$	$F(s)$
$\delta(t)$	$\Leftrightarrow$	1
$U(t)$	$\Leftrightarrow$	$\frac{1}{s}$
$tU(t)$	$\Leftrightarrow$	$\frac{1}{s^2}$
$e^{-at}U(t)$	$\Leftrightarrow$	$\frac{1}{s+a}$
$[\sin at]\,U(t)$	$\Leftrightarrow$	$\frac{a}{s^2+a^2}$
$[\cos at]\,U(t)$	$\Leftrightarrow$	$\frac{s}{s^2+a^2}$
$[e^{-at}\sin bt]\,U(t)$	$\Leftrightarrow$	$\frac{b}{(s+a)^2+b^2}$
$[e^{-at}\cos bt]\,U(t)$	$\Leftrightarrow$	$\frac{s+a}{(s+a)^2+b^2}$
$te^{-at}U(t)$	$\Leftrightarrow$	$\frac{1}{(s+a)^2}$

9.3 Laplace Transform rule

- Transform of Derivatives :

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

- Transform of Derivative of order n :

$$\mathcal{L}\{f^n(t)\} = s^n F(s) - \sum_{k=0}^{n-1} s^{n-1-k} f^k(0)$$

- Transform of an Integral :

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$$

- Derivative of Transforms

$$\begin{aligned} F'(s) &= \mathcal{L}\{-tf(t)\} \\ \text{and} \qquad \frac{\mathrm{d}^n}{\mathrm{d}s^n}F(s) &= (-1)^n \mathcal{L}\{t^n f(t)\} \end{aligned}$$

- Shift in the time-domain function :

$$\mathcal{L}\{f(t-t_0)U(t-t_0)\} = e^{-st_0}F(s)$$

- Shift in the s-domain function :

$$\mathcal{L}\{e^{-\alpha t}f(t)\} = F(s+\alpha)$$

- Initial Value Theorem :

$$\lim_{s\to\infty} sF(s) = f(0^+) = \lim_{t\to 0^+} f(t)$$

- Final Value Theorem (FVT) :

$$\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t)$$

  FVT is applicable only if the signal is a *finite constant* value, i.e. $\lim_{t\to\infty} f(t) =$ *constant*

Chapter 10

Fourier Series

The Fourier series corresponding to a periodic function $f(x)$ of period 2π is

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where for the range $-\pi$ to π

$$\begin{aligned}
a_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx \\
a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx \qquad (n = 1, 2, 3, \ldots) \\
b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx \qquad (n = 1, 2, 3, \ldots)
\end{aligned}$$

For functions of any period $2L$, the Fourier series is

$$\begin{aligned}
f(x) &= a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right) \\
a_0 &= \frac{1}{2L} \int_{-L}^{L} f(x)\, dx \\
a_n &= \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx \qquad (n = 1, 2, 3, \ldots) \\
b_n &= \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx \qquad (n = 1, 2, 3, \ldots)
\end{aligned}$$