EE3331C: Modeling Electrical Circuits

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Physical laws

1. Two basic laws:

   - conservation of charge: Kirchhoff's current law
   - conservation of energy: Kirchhoff's voltage law

2. Voltage-current relationship:

   - Resistor: $v(t) = Ri(t)$
   - Capacitor: $v_c(t) = \frac{1}{C}\int i(\tau)d\tau$
     the capacitance C of a capacitor is a measure of how much charge can be stored for a given voltage difference across the element, with units of charge per volt (i.e. farad (F)).
   - Inductor: $v_L(t) = L\frac{di(t)}{dt}$
     the unit of inductance L is the henry (H) which is one volt-second per ampere.

   The resistor, capacitor and inductor voltage-current relationships together with the conservation of charge and conservation of energy can be used to obtain circuit models.

Circuit anaylsis with impedances

1. Impedance:

   - idea: resistance resists/'*impedes*' current flow: $\frac{v}{i} = R$
   - capacitance and inductance elements also impede the flow of current
   - impedance is defined as the ratio of a voltage transform $V(s)$ to a current transform $I(s)$, i.e. $Z(s) = \frac{V(s)}{I(s)}$ (or the transfer function from current, $I(s)$ to voltage, $V(s)$).
   - note that in time domain, v and i are related by convolution, i.e. $v = z * i$.
   - impedance of resistor is thus: $Z_R(s) = R$.

- impedance of capacitor:

$$\begin{aligned} v(t) &= \frac{1}{C}\int_0^t i(\tau)d\tau \\ \text{LT gives} \quad V(s) &= \frac{1}{Cs}I(s) \qquad \text{with zero initial voltage} \\ \Rightarrow \quad Z(s) &= \frac{V(s)}{I(s)} = \frac{1}{Cs} \end{aligned}$$

- impedance of inductor:

$$\begin{aligned} v(t) &= L\frac{di(t)}{dt} \\ \text{LT gives} \quad V(s) &= LsI(s) \qquad \text{with zero initial current} \\ \Rightarrow \quad Z(s) &= \frac{V(s)}{I(s)} = Ls \end{aligned}$$

2. Series and parallel impedances:

- concept of impedance useful, individual elements can be combined in series or parallel law;
- key assumption, initial conditions for inductor and capacitor are zero;
- two or more impedances are in series if they have the *same current*, the total impedance is the sum of the individual impedance:

$$Z(s) = Z_1(s) + \ldots + Z_n(s)$$

e.g. the series RLC circuit, the equivalent impedance is given by

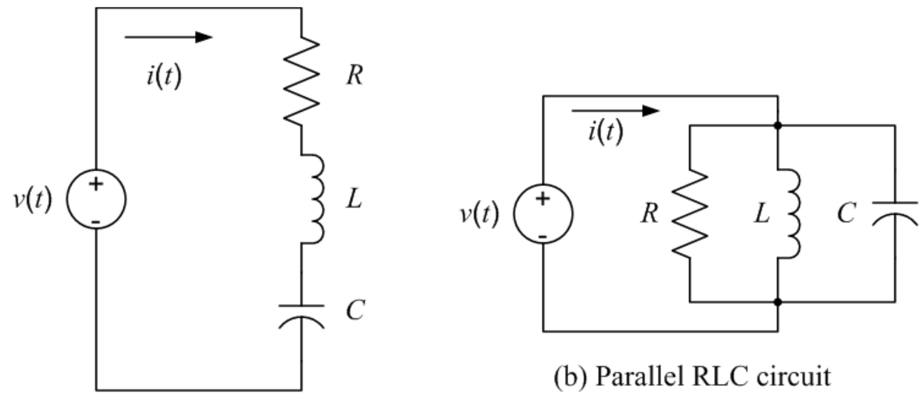
$$Z(s) = R + sL + \frac{1}{sC} = \frac{LCs^2 + RCs + 1}{sC}$$

- two or more impedances are in parallel if they have the *same voltage difference* across them, their impedances combine by the reciprocal rule:

$$\frac{1}{Z(s)} = \frac{1}{Z_1(s)} + \ldots + \frac{1}{Z_n(s)}$$

e.g. the parallel RLC circuit, the equivalent total impedance is

$$\begin{aligned} \frac{1}{Z(s)} &= \frac{1}{R} + \frac{1}{sL} + \frac{1}{1/(Cs)} = \frac{RLCs^2 + Ls + R}{RLs} \\ Z(s) &= \frac{RLs}{RLCs^2 + Ls + R} \end{aligned}$$

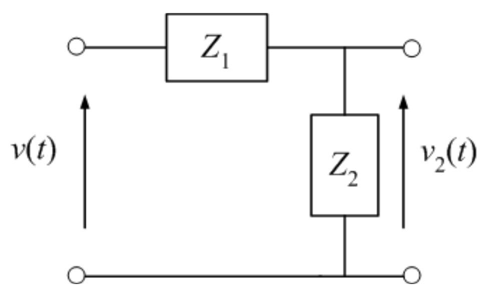


(a) Series RLC circuit

(b) Parallel RLC circuit

3. Voltage divider rule:

$$V_2(s) = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} V(s)$$



4. Current divider rule:

$$I_1(s) = \frac{Z_T(s)}{Z_1(s) + Z_T(s)} I(s)$$